

On neutrino masses via CPT violating Higgs interaction in the Standard Model

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Abstract

The Lorentz invariant CPT violation by using non-local interactions is naturally incorporated in the Higgs coupling to neutrinos in the Standard Model, without spoiling the basic $SU(2)_L \times U(1)$ gauge symmetry. The neutrino–antineutrino mass splitting is thus realized by the mechanism which was proposed recently, assuming the neutrino masses to be predominantly Dirac-type in the Standard Model.

1 Introduction

The CPT symmetry is a fundamental symmetry of local field theory defined in Minkowski space-time [1]. However, the possible breaking of CPT symmetry has also been discussed. One of the logical ways to break CPT symmetry is to make the theory non-local by preserving Lorentz symmetry, while the other is to break Lorentz symmetry itself. Lorentz symmetry breaking scheme has been mainly discussed in the past [2], but a possible mechanism to break CPT symmetry in a Lorentz invariant manner has also been proposed [3] (see also [4]). We then presented an explicit non-local Lagrangian model which induces the particle antiparticle mass splitting in a Lorentz invariant manner [5],

$$\begin{aligned}
 S = & \int d^4x \left\{ \bar{\psi}(x) i\gamma^\mu \partial_\mu \psi(x) - m \bar{\psi}(x) \psi(x) \right. \\
 & - \int d^4y [\theta(x^0 - y^0) - \theta(y^0 - x^0)] \delta((x - y)^2 - l^2) \\
 & \left. \times [i\mu \bar{\psi}(x) \psi(y)] \right\},
 \end{aligned} \tag{1}$$

which is Lorentz invariant and hermitian. For the real parameter μ , the third term has $C = CP = CPT = -1$ and thus no symmetry to ensure the equality of particle

and antiparticle masses. The parameter l has dimension of length, and the mass dimension of the parameter μ is $[M]^3$.

The free equation of motion for the fermion in (1) is given by

$$\begin{aligned} i\gamma^\mu \partial_\mu \psi(x) &= m\psi(x) \\ +i\mu \int d^4y [\theta(x^0 - y^0) - \theta(y^0 - x^0)] \delta((x - y)^2 - l^2) \psi(y). \end{aligned} \quad (2)$$

By inserting an ansatz for the possible solution $\psi(x) = e^{-ipx}U(p)$, we have

$$\not{p}U(p) = mU(p) + i\mu[f_+(p) - f_-(p)]U(p), \quad (3)$$

where $f_\pm(p)$ is the Lorentz invariant "form factor" defined by

$$f_\pm(p) = \int d^4z_1 e^{\pm ipz_1} \theta(z_1^0) \delta((z_1)^2 - l^2), \quad (4)$$

which are inequivalent for the time-like p due to the factor $\theta(z_1^0)$; this $f_\pm(p)$ is mathematically related to the two-point Wightman function for a free scalar field [5] and thus expected to be well-defined at least as a distribution. By assuming a time-like p , we go to the frame where $\vec{p} = 0$. Then the eigenvalue equation for the mass is given by

$$p_0 = \gamma_0 \left[m - 4\pi\mu \int_0^\infty dz \frac{z^2 \sin[p_0 \sqrt{z^2 + l^2}]}{\sqrt{z^2 + l^2}} \right], \quad (5)$$

where we used the explicit formula

$$f_\pm(p^0) = 2\pi \int_0^\infty dz \frac{z^2 e^{\pm ip^0 \sqrt{z^2 + l^2}}}{\sqrt{z^2 + l^2}}. \quad (6)$$

This eigenvalue equation under $p_0 \rightarrow -p_0$ becomes

$$p_0 = \gamma_0 \left[m + 4\pi\mu \int_0^\infty dz \frac{z^2 \sin[p_0 \sqrt{z^2 + l^2}]}{\sqrt{z^2 + l^2}} \right], \quad (7)$$

which is not identical to the original equation in (5). This causes the mass splitting of particle and antiparticle in the sense of Dirac, even if all C , CP and CPT symmetries are broken in the present model. See Ref. [5] for further details.

From the point of view of particle phenomenology, there is a strong interest in the possible mass splitting between the neutrino and associated antineutrino [6, 7, 8]. The purpose of the present letter is to discuss the application of the above mass splitting mechanism to Dirac-type neutrinos in the Standard Model.

2 Beyond the Standard Model

In the original Standard Model [9] the neutrinos are assumed to be massless, but recent experiments indicate non-vanishing neutrino masses. We thus go beyond the original Standard Model by including massive neutrinos.

We study a one-generation model of leptons to explain the essence of the mechanism. We consider a minimal extension of the Standard Model by incorporating the right-handed neutrino:

$$\psi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad \psi_R = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix} \quad (8)$$

and the part of the Standard Model Lagrangian relevant to our discussion is given by

$$\begin{aligned} \mathcal{L} = & \bar{\psi}_L i\gamma^\mu (\partial_\mu - igT^a W_\mu^a - i\frac{1}{2}g'Y_L B_\mu) \psi_L \\ & + \bar{e}_R i\gamma^\mu (\partial_\mu + ig'B_\mu) e_R + \bar{\nu}_R i\gamma^\mu \partial_\mu \nu_R \\ & - \left[\frac{\sqrt{2}m_e}{v} \bar{e}_R \phi^\dagger \psi_L + \frac{\sqrt{2}m_D}{v} \bar{\nu}_R \phi_c^\dagger \psi_L + \frac{m_R}{2} \nu_R^T C \nu_R \right] + h.c. \end{aligned} \quad (9)$$

with $Y_L = -1$, and the Higgs doublet and its $SU(2)$ conjugate:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \phi_c \equiv i\tau_2 \phi^* = \begin{pmatrix} \bar{\phi}^0 \\ -\phi^- \end{pmatrix}. \quad (10)$$

The operator C stands for the charge-conjugation matrix for spinors. The term with m_R in the above Lagrangian is the Majorana mass term for the right-handed neutrino [10].

We take the above Lagrangian as a *low-energy effective theory* and apply to it the naturalness argument of 't Hooft [11]. We first argue that the choice $m_D^2 \gg m_R^2$ is natural, since by setting $m_R = 0$ one recovers an enhanced fermion number symmetry in (9) [12, 13, 14]. We then argue that $m_e \gg m_D$ is also natural, since by setting $m_D = m_R = 0$ one finds an enhanced symmetry $\nu_R(x) \rightarrow \nu_R(x) + \xi_R$, with constant ξ_R , in the Lagrangian (9) [15]. Thus, our basic assumption in the present letter is $m_e \gg m_D \gg m_R$, namely, the so-called pseudo-Dirac scenario [12], and in the explicit analysis below we adopt the Dirac limit $m_R = 0$ for simplicity.

Our next observation is that the combination

$$\phi_c^\dagger(x) \psi_L(x) \quad (11)$$

is invariant under the full $SU(2)_L \times U(1)$ gauge symmetry. One may thus add a hermitian non-local Higgs coupling, which is analogous to the last term in (1), to the Lagrangian (9),

$$\mathcal{L}_{CPT}(x) = -i \frac{2\sqrt{2}\mu}{v} \int d^4y \delta((x-y)^2 - l^2) \theta(x^0 - y^0) \{ \bar{\nu}_R(x) (\phi_c^\dagger(y) \psi_L(y)) - (\bar{\psi}_L(y) \phi_c(y)) \nu_R(x) \}, \quad (12)$$

without spoiling the basic $SU(2)_L \times U(1)$ gauge symmetry. In the unitary gauge, $\phi^\pm(x) = 0$ and $\phi^0(x) \rightarrow (v + \varphi(x))/\sqrt{2}$, the neutrino mass term (with $m_R = 0$) becomes in terms of the action

$$\begin{aligned} S_{\nu\text{mass}} &= \int d^4x \left\{ -m_D \bar{\nu}(x) \nu(x) \left(1 + \frac{\varphi(x)}{v} \right) \right. \\ &\quad - i\mu \int d^4y \delta((x-y)^2 - l^2) \theta(x^0 - y^0) \\ &\quad \times \left[\bar{\nu}(x) \left(1 + \frac{\varphi(y)}{v} \right) (1 - \gamma_5) \nu(y) - \bar{\nu}(y) \left(1 + \frac{\varphi(x)}{v} \right) (1 + \gamma_5) \nu(x) \right] \Big\} \\ &= \int d^4x \left\{ -m_D \bar{\nu}(x) \nu(x) \left(1 + \frac{\varphi(x)}{v} \right) \right. \\ &\quad - i\mu \int d^4y \delta((x-y)^2 - l^2) [\theta(x^0 - y^0) - \theta(y^0 - x^0)] \bar{\nu}(x) \nu(y) \\ &\quad + i\mu \int d^4y \delta((x-y)^2 - l^2) \bar{\nu}(x) \gamma_5 \nu(y) \\ &\quad - i \frac{\mu}{v} \int d^4y \delta((x-y)^2 - l^2) \theta(x^0 - y^0) \\ &\quad \times [\bar{\nu}(x) (1 - \gamma_5) \nu(y) - \bar{\nu}(y) (1 + \gamma_5) \nu(x)] \varphi(y) \Big\}, \quad (13) \end{aligned}$$

where we have changed the naming of integration variables $x \leftrightarrow y$ in some of the terms and used $\theta(x^0 - y^0) + \theta(y^0 - x^0) = 1$.

When one looks at the mass terms in (13) without the Higgs φ coupling, the first two terms are identical to the two terms in (1) but an extra parity-violating non-local mass term appears, which adds an extra term $-i\mu\gamma_5 g(p^2)$ to m in the mass eigenvalue equations in (5) and (7); here $g(p^2) = \int d^4z_1 e^{ipz_1} \delta((z_1)^2 - l^2)$. This extra term is C and CPT preserving and does not contribute to the mass splitting. Since we are assuming that CPT breaking terms are very small, we may solve the mass eigenvalue equations iteratively by assuming that the terms with the parameter μ are much smaller than $m = m_D$. We then obtain the mass eigenvalues of the neutrino

and antineutrino at

$$m_{\pm} \simeq m_D - i\mu\gamma_5 g(m_D^2) \pm 4\pi\mu \int_0^\infty dz \frac{z^2 \sin[m_D \sqrt{z^2 + l^2}]}{\sqrt{z^2 + l^2}}, \quad (14)$$

where we have used the upper two (positive) components of the matrix γ_0 in (5) and (7). The parity violating mass $-i\mu\gamma_5 g(m_D^2)$ is now transformed away by a suitable global chiral transformation without modifying the last term in (14) to the order linear in the small parameter μ . In this way, the neutrino and antineutrino mass splitting is incorporated in the Standard Model by a Lorentz invariant non-local CPT breaking mechanism, without spoiling the $SU(2)_L \times U(1)$ gauge symmetry. The Higgs particle φ itself has a tiny C -, CP - and CPT -violating coupling in (13).

3 Discussion

We have assumed Dirac-type neutrinos, but this may not be unnatural in the present context since the notion of antiparticle is best defined for a Dirac particle. In other words, if the neutrino–antineutrino mass splitting is confirmed by experiments, it would imply that neutrinos are Dirac-type particles rather than Majorana-type particles. Also, our identification of the neutrino mass terms as the origin of the possible CPT breaking may be natural if one recalls that the mass terms of the neutrinos are the known origin of new physics beyond the original Standard Model. The remaining couplings of the Standard Model are very tightly controlled by the $SU(2)_L \times U(1)$ gauge symmetry, and one can confirm that only the neutrino mass terms allow the present non-local gauge invariant couplings without introducing Wilson-line type gauge interactions. (An analysis of the scheme with Wilson-lines, which goes beyond the conventional local gauge principle, will be given elsewhere [16].)

To apply our scheme to the analysis of neutrino phenomenology including neutrino oscillation, we need to generalize the scheme to the three generations of neutrinos. We consider that the generalization including the neutrino mixing does not present a difficulty of basic principle, although a detailed analysis of the three generations of neutrinos and the possible choice of the parameters l and μ in our scheme is required. It could be that our scheme needs to be generalized by introducing more free parameters to apply it to realistic particle phenomenology. Thus, our model may provide an indirect support for the speculation on the possible mass splitting between the neutrino and antineutrino [6].

If such a splitting will indeed be observed by future experiments, the presented pseudo-Dirac scheme could be considered as an economical alternative to seesaw mechanism [10], where at the same time an explanation for the mass splitting between the particle and its antiparticle is provided.

Finally, we would like to discuss some basic field theoretical issues related to the non-local couplings in our scheme. As for the quantization of the theory non-local in time, for which the notion of canonical momentum is ill-defined, our suggestion is to use the path integral on the basis of Schwinger's action principle. This path integral is based on the equation of motion and provides correlation functions which agree with the ordinary quantum mechanical correlations for local theory; the canonical structure is recovered later by means of Bjorken–Johnson–Low prescription [17]. For non-local theory, this scheme provides a possible generalization and provides a convenient scheme for the treatment of non-local terms as small perturbation.

It is also well-known that a theory non-local in time generally spoils unitarity. In our scheme we treat small non-local couplings in (13) in the lowest order of perturbation, for which the effects of the violation of unitarity are expected to be minimal. However, we have the neutrino propagator

$$\langle T^* \nu(x) \bar{\nu}(y) \rangle = \int \frac{d^4 p}{(2\pi)^4} e^{-ip(x-y)} \frac{i}{\not{p} - m_D + i\epsilon + i\mu\gamma_5 g(p^2) - i\mu[f_+(p) - f_-(p)]}, \quad (15)$$

which includes the effects of non-local terms. In the pole approximation this propagator gives a sensible result in (14), but it may lead to difficulties in the off-shell domain. Alternatively, the *CPT*-violating terms in the presented scheme as such could be regarded as the low-energy limit of a more basic theory or coming from some higher-dimensional theories [14], whose compactification would lead to non-local interactions, and thus the unitarity issue may be postponed to future study. Otherwise, it is very gratifying that the basic $SU(2)_L \times U(1)$ gauge symmetry together with Lorentz symmetry are exactly preserved by our non-local *CPT* violation. We can thus avoid the appearance of negative norm in the gauge sector if one applies gauge invariant and Lorentz invariant regularization.

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